ISSN 2229-5518

Flow of a non-Newtonian Fluid through a Vertical Channel with Porous Medium

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Abstract: The effect of magnetic field on the flow of non-Newtonian fluids through different channels containing porous medium has very important role for industries in designing MHD generators and other devices. They need a deep knowledge of MHD flow through different channels. In this paper an attempt has been made to find the effect of magnetic field on the flow of a non-Newtonian fluid through vertical channel filled with porous medium under the transversely applied magnetic field. The result clearly indicates that the velocity profile of fluid can be easily controlled by applying magnetic field.

Introduction

Magnetohydrodynamic flow of fluid has a vital role in science and technology and particularly in industries. Electromagnetic pumps, hydromagnetic generators and energy storage devices require thorough knowledge of this field and practically, the flow occurs through porous medium and the fluid existing in nature are generally non-Newtonian therefore this study concentrates on non-Newtonian fluid flowing in a magnetic field through vertical plate channel completely filled with porous medium.

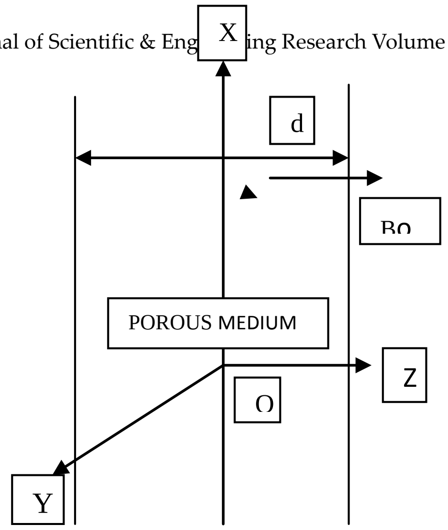
The objective of this paper is to find the effect of magnetic field on velocity profile of the fluid flowing through a porous channel.

Many researchers have made valuable contributions in this field. Nigam and Singh [1960]; Sondalgekar and Bhat [1971];Attia and Kob,[1996] studied the effect of transversely applied magnetic field on convection flows of an electrically conducting fluid. Rapits et al., [1982] studied hydromagnetic free convection flow through porous medium between two parallel plates. Singh, Garg and Bansal [2014] studied Hall current effect in visco-elastic MHD oscillatory convective flow through a porous medium in a vertical channel with heat radiation.

Since, the fluid considered is non- Newtonian therefore additional factor has been introduced to study the problem under consideration.

Formulation of the problem:

Consider an unsteady MHD flow of non-Newtonian incompressible fluid through an electrically conducting porous medium existing between two infinite vertical plates in the presence of hall currents and thermal radiation. The two plates are considered at a distance d apart. Considering the Cartesian co-ordinate system with x'-axis fixed vertically upward along the centre line of the channel. The z' –axis is assumed perpendicular to the planes of the plates along which a strong transverse magnetic field is applied as shown in following figure.



In this paper, non-Newtonian factor has been introduced with magnetic field term to study the behavior of fluid velocity. All the physical quantities except the pressure depend only on z' and t' only. Let (u', v', w') be the components of velocity in directions (x', y', z') respectively. Since the plates are non-porous, therefore equations of continuity ($\nabla \cdot V = 0$) on integration gives $w' = 0$. Also the gauss's law of magnetism equation $div\overline{\mathrm{B}} = 0$ for the magnetic field gives $\overline{\mathrm{B}} = \left(B_x{'}, B_y{'}, B_z{'}\right), B_z{'} = B_\circ$ (constant).

It is assumed no applied and polarization voltage exists.

Electric field $\overline{E} = 0$.

$$\bar{j} + \frac{\omega_e \tau_e}{\mathrm{B}_\circ}(J \times \mathrm{B}) = \sigma(V \times \mathrm{B}) \quad \text{(Ohm's law)} \tag{1}$$

$\left(j_x{'}, j_y{'}, j_z{'}\right)$ are the components of electric current density $\bar{j}$, the conservation of electric charge div $\bar{j}$ =0 gives $\left(j_z{'} = 0\right)$ (constant) (2)

$J_z{'} = 0$ at the plates and hence zero everywhere in the fluid. Under the assumptions that the electron pressure (for weakly ionized gas) the thermo electric pressure, ion slip and the external electric field arising due to polarization of charges is negligible.

Using equation (2),

Equation (1) becomes,

$$j_x{'} + \omega_e \tau_e j_y{'} = \sigma \mathrm{B}_\circ \bar{v}' \tag{3}$$

$$j_y{'} + \omega_e \tau_e j_x{'} = \sigma \mathrm{B}_\circ \bar{u}' \tag{4}$$

Solving equation (3) and (4) for $j_x{'}$ *and* $j_y{'}$

$$j_x' = \frac{\sigma B_\circ}{(1+H^2)}\left(Hu' + v'\right) \tag{5}$$

And

$$j_y' = \frac{\sigma B_\circ}{(1+H^2)}\left(Hv' + u'\right) \tag{6}$$

Where, $H = \omega_e \tau_e$ (Hall parameter)

Under the Boussinesq approximation momentum equation

$$\rho\left[\frac{\partial \overline{V}}{\partial t'} + \left(\overline{V}\cdot\nabla\right)V\right] = -\nabla p + J\times B + \mu\nabla^2\overline{V} - \frac{\mu}{K'}V + g\beta\left(T' - T_\circ\right) + \nabla\cdot\exists$$ in Cartesian components reduce to

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu_1\frac{\partial^2 u'}{\partial z'^2} + \nu_2\frac{\partial^3 u'}{\partial z'^2\partial t'} + \frac{\sigma B_\circ^{\,2}\left(Hv' - u'\right)}{\rho\left(1+H^2\right)\left(1+c^2\right)} - \frac{\nu_1 u'}{K'} + g\beta\left(T' - T_\circ\right), \tag{7}$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial y'} + \nu_1\frac{\partial^2 v'}{\partial z'^2} + \nu_2\frac{\partial^3 v'}{\partial z'^2\partial t'} + \frac{\sigma B_\circ^{\,2}\left(Hu' - v'\right)}{\rho\left(1+H^2\right)\left(1+c^2\right)} - \frac{\nu_1 v'}{K'}, \tag{8}$$

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q'}{\partial z'} \text{ (energy equation)} \tag{9}$$

Corresponding boundary conditions are:

$$u' = v' = 0 \quad ; \quad T' = T_\circ \quad \text{at} \quad z' = -\frac{d}{2}, \tag{10}$$

$$u' = v' = 0,$$

$$T' = T_\circ + \left(T_w - T_\circ\right)\cos\omega' t' \quad \text{at} \quad z' = -\frac{d}{2}, \tag{11}$$

Fluid is optically thin with relatively low density and the last term in equation (9),

$$\frac{\partial q'}{\partial z'} = 4\alpha^2\left(T' - T_\circ\right) \tag{12}$$

Stand for radiative heat flux, where

$$\alpha^2 = \int_0^\infty k_{\lambda n}\frac{\partial e_{b\lambda}}{\partial T}\,d\lambda. \tag{13}$$

Where $k_{\lambda n}$: absorption coefficient at the walls.

$e_{b\lambda}$: Plank's function.

ISSN 2229-5518

Non-dimensional quantities are:

$$\eta=\frac{z'}{d}, x=\frac{x'}{d}, y=\frac{y'}{d}, u=\frac{u'}{U}, v=\frac{v'}{U}, T=\frac{T'-T_{\circ}}{T_w-T_{\circ}}$$

$$t=\frac{t'U}{d}, \omega=\frac{\omega' d}{U}, p=\frac{p'}{\rho U^2}$$

Using non-dimensional quantities and solving equations (7) to (9) and using equation (11),

$$R_e\frac{\partial u}{\partial t}=-R_e\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial \eta^2}+\gamma\frac{\partial^3 u}{\partial \eta^2\partial t}+\frac{M^2(Hv-u)}{(1+H^2)(1+c^2)}-\frac{1}{K}u+G_rT. \tag{14}$$

$$R_e\frac{\partial v}{\partial t}=-R_e\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial \eta^2}+\gamma\frac{\partial^3 v}{\partial \eta^2\partial t}+\frac{M^2(Hu-v)}{(1+H^2)(1+c^2)}-\frac{1}{K}v. \tag{15}$$

$$P_e\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial \eta^2}-N^2T. \tag{16}$$

The corresponding transformed boundary conditions are:

$$u=v=0, T=0 \quad \text{at} \quad \eta=-\frac{1}{2} \tag{17}$$

$$u=v=0, T=\cos\omega t \quad \text{at} \quad \eta=\frac{1}{2} \tag{18}$$

Where,

$R_e=\dfrac{Ud}{\nu_1}$: Reynold's number,

$K=\dfrac{K'}{d^2}$: The permeability of porous medium.

$M=\mathrm{B}_{\circ}d\sqrt{\dfrac{\sigma}{\rho\nu_1}}$: Hartmann number,

$\gamma=\dfrac{\nu_2R_e}{d^2}$: Visco-elastic parameter,

$H=\omega_e\tau_e$: Hall parameter,

$G_r=\dfrac{g\beta d^2(T_W-T_{\circ})}{\nu_1U}$: Grashof number,

$N=\dfrac{2\alpha d}{\sqrt{k}}$: Radiation parameter,

$P_e = \dfrac{\rho C_p dU}{k}$: Peclet number,

Following Singh and Pathak [2013], for the oscillatory internal flow considered we shall assumed the fluid flows only under the influence of a non-dimensional pressure gradient oscillating in the direction of x-axis which is the form

$$-\frac{\partial p}{\partial x} = A\cos\omega t \quad \text{and} \quad -\frac{\partial p}{\partial y} = 0 \tag{19}$$

Solution of the problem:

Combining equations (14) and (15), introduce a complex function F=u+iv and using equation (19),

$$R_e \frac{\partial p}{\partial x} - G_r T = \gamma \frac{\partial^3 F}{\partial \eta^2 \partial t} + \frac{\partial^2 F}{\partial \eta^2} - R_e \frac{\partial F}{\partial t} - \left(\frac{M^2 (1+iH)}{(1+H^2)(1+c^2)} + K^{-1} \right) F \tag{20}$$

The boundary conditions (17) and (18) in complex form can be written as:

$$F = 0, T = 0 \quad \text{at} \quad \eta = -\frac{1}{2,} \tag{21}$$

$$F = 0, T = \cos\omega t \quad \text{at} \quad \eta = \frac{1}{2,} \tag{22}$$

Solving equations (16) and (20) using boundary conditions (21) and (22),

Assume complex form the solution of the problem as:

$$F(\eta,t) = F_\circ(\eta) e^{i\omega t}, T(\eta,t) = \theta_\circ(\eta) e^{i\omega t}$$
$$and$$
$$-\frac{\partial p}{\partial x} = A e^{i\omega t} \tag{23}$$

The boundary conditions (21) and (22) become:

$$F = 0, \theta_\circ = 0 \; \eta = -\frac{1}{2,} \tag{24}$$

$$F = 0, \theta_\circ = 1 \; \eta = \frac{1}{2,} \tag{25}$$

Putting the value of equation (23) in equations (16) and (20),

$$l^2 F_\circ'' - Q^2 F_\circ = -AR_e - G_r \theta_\circ \tag{26}$$

and

$$\theta_{\circ}'' - n^2\theta_{\circ} \tag{27}$$

Where,

$$Q^2 = \left\{ \frac{M^2(1+iH)}{(1+H^2)(1+c^2)} + K^{-1} + i\omega R_e \right\}$$

$$l^2 = (1 + i\omega\gamma)$$

$$n^2 = N^2 + i\omega P_e$$

The ordinary differential equation (26) and (27) are solved under the boundary conditions (24) and (25), the solution of the problem is obtained:

$$F = \left[\frac{AR_e}{Q^2}\left\{ 1 - \frac{\cosh \frac{Q}{l}\eta}{\cosh \frac{Q}{2l}} \right\} + \frac{G_r}{(l^2n^2 + Q^2)}\left\{ \frac{\sinh \frac{Q}{l\left(\eta + \frac{1}{2}\right)}}{\sinh \frac{Q}{l}} - \frac{\sinh n\left(\eta + \frac{1}{2}\right)}{\sinh n} \right\} \right] e^{i\omega t} \tag{28}$$

$$T(\eta, t) = \left[\frac{\sinh\left(\eta + \frac{1}{2}\right)}{\sinh n} \right] e^{i\omega t} \tag{29}$$

To observe the velocity profile with respect to applied magnetic field, suitable values for different parameters are chosen to plot the curve between magnetic field and velocity profile for different values of non-Newtonian factors.

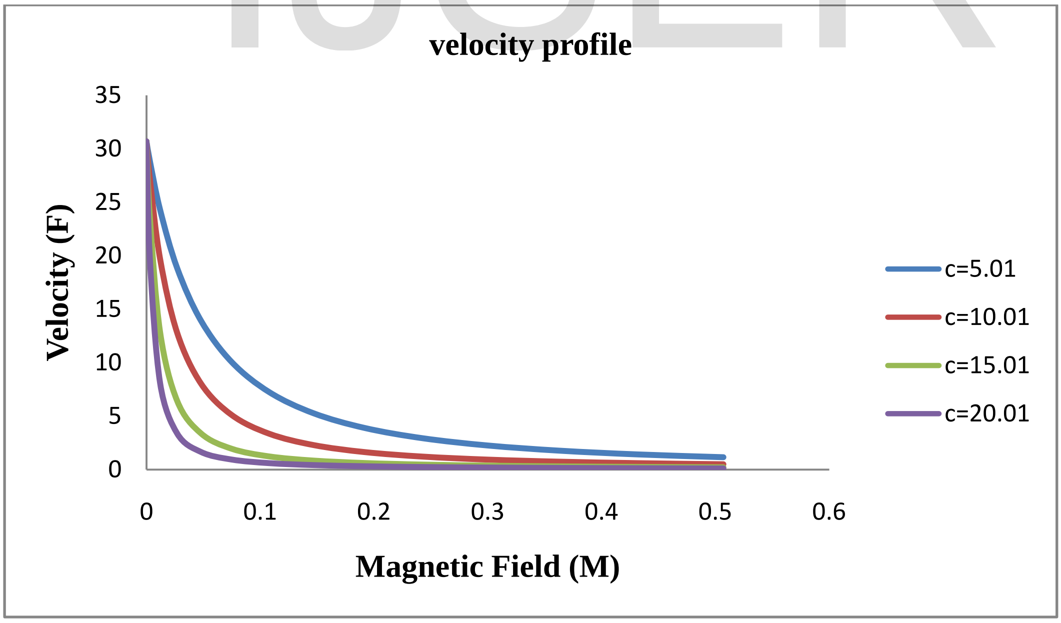
Result and Discussion

Table : Magnetic field and Velocity profile.

The values of parameters are:

(H =1, K=0.1, N=1, A= 4, $R_e = 0.5, P_e = 1, G_r = 5$, ϒ=0.1)

		Velocity of fluid			
S. No.	Magnetic field (M)	non-Newtonian (c)=5.01	non-Newtonian (c) =10.01	non-Newtonian (c)=15.01	non-Newtonian (c)=20.01
1	0.000	30.704964	30.7049645	30.70496445	30.70496445
2	0.003	29.730819	27.1827384	19.76842816	12.88359629
3	0.012	27.081841	19.7033827	8.095130193	3.386722800
4	0.027	23.419231	12.7941786	3.379734699	1.225758833
5	0.048	19.461496	8.03197549	1.648711320	0.605396884
6	0.075	15.737196	5.09926835	0.939683846	0.362987652
7	0.108	12.520508	3.34559540	0.604168979	0.243794861
8	0.147	9.8874325	2.28723747	0.422974363	0.175752663
9	0.192	7.8011381	1.63113482	0.314156087	0.132963431
10	0.243	6.1781810	1.21016414	0.243345900	0.104205453
11	0.300	4.9267434	0.92992069	0.194442543	0.083913117
12	0.363	3.9640760	0.73643568	0.159125395	0.069046647
13	0.432	3.2221064	0.59817967	0.132726382	0.057823561
14	0.507	2.6474635	0.49624235	0.112446442	0.049139896



It is indicated from the graph that the velocity decreases with increase of magnetic field for every value of non-Newtonian factor, the form of each curve remains the same, initially all start from a fixed value of velocity, following a hyperbolic character, become parallel to the x-axis. However, the curvature is highly affected by the non-Newtonian values the curve has more curvature for greater values of non-Newtonian factor. It appears from the derived relation for velocity that non-Newtonian factor is an important component in controlling the velocity as it is associated with many terms present in the relation. Also the hyperbolic functions of various parameters play any important role in controlling the velocity. The transverse magnetic field on an electrically conducting fluid creates a resistive force or drag force which looses the velocity and this is also relevant from the presence of such term in the relation. The presence of other parameters cannot be ignored

Hence motion of such fluids can be controlled easily by magnetic field and non-Newtonian factor and this can find applications in industries.

References

[1] Attia, H.A., 2004, "Unsteady Hartmann flow of a viscielastic fluid considering the Hall Effect." Canadiam, J. Phy82pp. 127
[2] Chaudhary, R., Bhattacharjee, H. K., and Paban, D., 2013,"Heat and Mass transfer past a vertical porous plat in presence of Hall current and radiation," Int J. fluid Eng., (5) pp 39-55.
[3] Singh, K. K., 1977, "Unsteady Flow of Conducting Dusty Viscous Liquid in an Annulus," Acta Ciecia Indica,(3)pp264.
[4] Lamb, Sir Horace,1993, "Hydrodynamics," Cambridge Mathematical Library.
[5] Singh, K.D., Garg, B.P., and Bansal, A.K., 2014, "Hall current effect on visco- elastic MHD oscillatory convective flow through a porous medium in a vertical channel with heat radiation," Proc Indian Nath Sci Acad., 82(2) pp333-343.
[6] Singh, P., and Gupta, C. B., 2005, "MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer," Ind. J. Theo. Phys., (2) pp. 111-112
[7] Sharma, P.R., and Singh, G., 2008, "Unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation," Int. J. Appl.Math.Mech., 4 (5), pp.1-8.